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REQUEST FOR FILING A CONTINUATION OR DIVISION OF AN INTERNATIONAL APPLICATION

DOCKET NUMBER	ANTICIPATED CLASSIFICATION OF THIS APPLICATION		PRIOR APPLICATION EXAMINER	ART UNIT
NOMA-110	CLASS	SUBCLASS		

Address to:  
Assistant Commissioner for Patents  
Washington, D.C. 20231

This is a request for filing a ☒ continuation -in-part; application under 37 CFR 1.53(b), of pending prior international application Number PCT97/ 15605, filed on 5 SEP 97 entitled METHOD AND APPARATUS

FOR ROBOT MOTION CONTROL

which designated the United States.

Note: 37 CFR 1.53(c) or (d) cannot be used to file a continuation or divisional application of an international application which has not entered the national stage.

CLAIMS	(1) FOR	(2) NUMBER FILED	(3) NUMBER EXTRA	(4) RATE	(5) CALCULATIONS
	TOTAL CLAIMS (37 CFR 1.16(c))	31 - 20 =	11	x \$ 18 =	\$ 198.00
	INDEPENDENT CLAIMS (37 CFR 1.16(b))	11 - 3 =	8	x \$ 78 =	624.00
	MULTIPLE DEPENDENT CLAIMS (if applicable) (37 CFR 1.16(d))			+ \$ =	
				BASIC FEE (37 CFR 1.16(a))	+ 760.
				Total of above Calculations =	\$ 1,582.00
	Reduction by 50% for filing by small entity (Note 37 CFR 1.9, 1.27, 1.28).				791.00
				TOTAL =	\$ 791.00

- Enclosed are the specification, claims and drawing(s).
- ☒ A statement to establish small entity status under 37 CFR 1.9 and 1.27 is enclosed. UNSIGNED.
- ☒ The Commissioner is hereby authorized to charge any fees which may be required under 37 CFR 1.16 and 1.17, or credit any overpayment to Deposit Account No. 12-1420. A duplicate copy of this sheet is enclosed.
- ☒ A check in the amount of \$ 791.00 is enclosed.
- ☒ Amend the specification by inserting before the first line the sentence: "This application is a ☒ continuation -in-part ☐ division of international application number PCT 97/ 15605, filed 5 SEPTEMBER 1997, (status, abandoned, pending, etc.)." which claims priority of U.S. provisional application 60/025,406, filed on 6 SEPTEMBER 1996."

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Burden Hour Statement: This form is estimated to take 0.5 hours to complete. Time will vary depending upon the needs of individual case. Any comments on the amount of time you are required to complete this form should be sent to the Chief Information Officer, Patent and Trademark Office, Washington, DC 20231. DO NOT SEND FEES OR COMPLETED FORMS TO THIS ADDRESS. SEND TO: Assistant Commissioner for Patents, Washington, DC 20231.

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(REQUEST FOR FILING A CONTINUATION OR DIVISION OF AN INTERNATIONAL APPLICATION, PAGE 2)

6. ☒ A declaration under 37 CFR 1.63 is enclosed. **UNSIGNED.**

7. ☒ Priority of foreign application number PCT/US97/15605, filed on 5 SEPTEMBER 1997 in U.S.R.O. is claimed under 35 U.S.C. 119(a) - (d), priority of U.S. provisional appln. 60/025,406, filed on 6 SEPTEMBER 1996 is claimed under 35 U.S.C. 119(e).

☐ The certified copy is enclosed.

8. ☐ A preliminary amendment is enclosed.

9. ☐ Also enclosed:

Address all future correspondence to: (May only be completed by applicant, or attorney or agent of record.)

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3-5-99

Date

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Signature

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Typed or printed name

**EXPRESS MAIL CERTIFICATE**

I hereby certify that this correspondence is being deposited with the United States Postal Service "Express Mail Service" Label No. EL186213 776 US, postage paid in an envelope, addressed to: Assistant Commissioner for Patents, Washington, DC 20231 on 5 March 1999

Dated: 5 March 1999 By: Douglas C. Limbach  
 LIMBACH & LIMBACH L.L.P.  
 Name

- ☐ Inventor(s)  
☐ Assignee of complete interest. Certification under 37 CFR 3.73(b) is enclosed.  
☒ Attorney or agent of record  
☐ Filed under 37 CFR 1.34(a)

Registration number if acting under 37 CFR 1.34(a).35,249.

# METHOD AND APPARATUS FOR MOBILE ROBOT MOTION CONTROL

This is a continuation-in-part of international application number  
5 PCT/US97/15605, filed on September 5, 1997, which claims a priority of U.S.  
provisional application 60/025,406, filed on September 6, 1996.

## Field of the Invention

The present invention relates generally to manned or unmanned  
10 omni-directional, normal-wheeled vehicles. More precisely, this invention involves a  
method and apparatus for controlling the motion of mobile base used as a vehicle.

## Background of the Invention

Mobile bases with intended applications in robotics and industrial automation  
require a great deal of movement flexibility in order to be fully utilized. Current robot  
15 mobile bases often employ a "synchro-drive" mechanism—a complex set of gears and  
pulleys that constrains the wheels to steer and to translate simultaneously. Since  
steering and translation movement is fully decoupled in this system, "steering in  
place" is possible. (Contrast this maneuverability with that of a car.) While this  
allows a good deal of maneuverability, its mechanical complexity makes it difficult  
20 and expensive to manufacture. Additionally, this type of mobile base has limited  
movement due to its fixed orientation (i.e. its inability to rotate).

In general, a rigid body constrained to move in a plane (i.e. a mobile base  
moving on the floor) has three degrees of freedom (DOFs) such as, movement in the x  
direction, movement in the y direction and rotation. Combining these three DOFs  
25 results in movement in any direction while simultaneously rotating.

A *holonomic* mobile base, has the ability move in this manner and change its  
motion at any time. By constructing a mobile base out of wheels that each have two  
degrees of freedom (one for steering, one for translation) all three DOFs are possible

under the proper control, and complex mechanisms found in synchro-drive mobile bases are no longer needed. Additionally, these 2-DOF wheels (2-DOFWs) can be easily integrated and manufactured as modular wheel assemblies. A complete description of one example of a robot base and modular wheel assembly is provided by U.S. Patent Application 09/134,241, by Holmberg et al., incorporated herein by reference.

It is also possible to increase the “caster” of a 2-DOFW by moving the translation axis behind the steering axis (Figure 1) and making it resemble a caster wheel. Unlike a 2-DOFW wheel with no caster (i.e. intersecting steering and translation axes), a mobile base constructed with caster 2-DOFWs is fully holonomic under the proper control.

Various problems arise, however, when trying to control a mobile base constrained to three DOFs with more than three controllable DOFs (e.g. a base with four 2-DOFWs has eight DOFs). A base with this many DOFs under improper control will certainly result in undesired motion and motor axes that “fight” each other. (i.e. consider two wheels facing opposite directions playing tug-of-war.)

This in turn creates wheel slippage, increased tire wear, increased power consumption, and more frequent mechanical problems.

It is considered important in robotics to be able to accurately assess the motion of the mobile base either through direct measurement or through estimation. This motion estimation can also be “summed-up” over time to create a “dead reckoned” position estimate of the mobile base with respect to fixed coordinates, which is also useful for autonomous tasks such as navigation. That is, consider a robot that wishes to navigate to a location (room) of which it knows the x-y coordinates. Motion estimation is also responsible for proper control of the mobile base, as will be described below. A control algorithm which minimizes wheel slippage will also allow for maximum motion estimation accuracy.

## Summary of the Invention

The present invention overcomes the deficiencies in the prior art described above, and provides motion control for a mobile robot base that is more accurate and more maneuverable Referring to Figure 3 and the Detailed Description of the

Preferred Embodiments, an overview of the inventive control sequence is provided in the following paragraph.

A supervisory controller reads the input vector from a host processor, and maps the input vector to the desired axis motion vector (i.e. the desired motion of each axis) by using the equations in Section 2 below. It then predicts if the axes are capable of the desired motion by calculating their control envelopes (i.e. the motion possible within one control cycle  $\Delta t$ .) as described in Section 3. If all axes are capable of the desired motion within one control cycle the desired axis motion vector is passed to the low-level controller. If one or more axes are incapable of the desired motion, a modified axis motion vector is calculated as described in Section 4 and passed to the low-level controller. The modified axis motion vector lies within the control envelopes of all  $2N$  axes while minimizing control error (i.e. the difference between the commanded input vector and the actual base motion.) The control algorithm then estimates the motion of the mobile base since the last control cycle using the technique described in Section 5. The estimated motion is then used to update the position and orientation  $[x_B, y_B, \psi_B]^T$  of the base coordinate frame in the fixed world coordinate frame as described also in Section 5. The updated position and orientation is then made available for the host processor to read. The control algorithm then repeats the whole process by beginning another control cycle.

## **Brief Description of the Drawings**

Figure 1A is a perspective view showing a mobile base wheel with no caster offset.

Figure 1B is a perspective view showing a mobile base wheel with a caster offset.

Figure 2 is a schematic diagram showing the flow of command information which controls the motion of the mobile base.

Figure 3 is a schematic diagram showing the steps of a method for controlling the motion of the mobile base.

Figure 4 is a perspective view schematically showing the layout of the mobile base wheels and the base and world coordinate systems.

## Detailed Description of the Preferred Embodiments

### Section 1 - Overview of the Control Algorithm

Referring to Table 1, a description of notations and variables used herein is provided.

5 **Table 1**

	$N$	Number of wheels
	$[x_{wi}, y_{wi}]^T$	Cartesian position of wheel attachment point of 2-DOFW $i$ to the base in base coordinates (m, m)
10	$[x'_{wi}, y'_{wi}]^T$	Cartesian position of wheel attachment point of 2-DOFW $i$ to the base in base coordinates as it has changed since the previous control cycle using the coordinates of the previous control cycle (m, m)
	$\theta_{mwi}$	Measured steer angle of 2-DOFW $i$ (rad)
	$\theta_{dwi}$	Desired steer angle of 2-DOFW $i$ (rad)
15	$\bar{m}_d$	Input vector (IV), or desired mobile base motion in base coordinates read from host processor (m/s, m/s, rad/s), (m/s <sup>2</sup> , m/s <sup>2</sup> , rad/s <sup>2</sup> ), or (N, N, Nm) dependant on desired implementation
20	$\bar{m}_x = [\dot{x}, \dot{y}, \dot{\psi}]^T$	Actual mobile base motion in base coordinates (m/s, m/s, rad/s)
	$\bar{m}_e = [\dot{x}_e, \dot{y}_e, \dot{\psi}_e]^T$	Estimated mobile base motion in base coordinates calculated as described in Section 5 (m/s, m/s, rad/s)
25	$\Delta m_e = [\Delta x_e, \Delta y_e, \Delta \psi_e]^T$	Estimated change in mobile base position in base coordinates calculated as described in Section 5 (m, m, rad)
	$\bar{m}_l(\lambda)$	Possible mobile base motion decomposed along parametric line with parameter $\lambda$ (m/s, m/s, rad/s) (Section 4)
	$\bar{m}_m$	Modified mobile base motion (m/s, m/s, rad/s)
30	$\bar{m}_a$	Axis motion vector (AMV) of length 2N. (Type of wheel (caster or no caster) and type of low level controller determines units)
	$\delta$	Small increment much less than 1.0 (0.1 nominal) (Section 4)
35	$[\dot{x}_{dwi}, \dot{y}_{dwi}]^T$	Desired velocity vector of attachment point of 2-DOFW $i$ to the base in base coordinates (m/s, m/s)

	$\dot{\mu}_{dWi}$	Desired velocity magnitude of 2-DOFW $i$ (m/s)
	$s_{dWi}$	Desired steering axis position for 2-DOFW $i$ (encoders)
	$s_{mWi}$	Measured steering axis position for 2-DOFW $i$ (encoders)
5	$\gamma s_{mWi}$	Measured steering axis position for 2-DOFW $i$ in the previous control cycle (encoders)
	$\Delta s_{mWi}$	Measured change in steering axis position for 2-DOFW $i$ since the previous control cycle (m)
10	$\dot{s}_{dWi}$	Desired steering axis velocity for 2-DOFW $i$ (encoders/s)
	$\dot{s}_{mWi}$	Measured steering axis velocity for 2-DOFW $i$ (encoders/s)
	$\dot{s}_{\max Wi}$	Maximum steering axis velocity for 2-DOFW $i$ (encoders/s)
15	$\ddot{s}_{\max Wi}$	Maximum steering axis acceleration for 2-DOFW $i$ (encoders/s <sup>2</sup> )
	$s_{lWi}$	Lower-bound steering axis position for 2-DOFW $i$ within $\Delta t$ time (encoders)
20	$s_{uWi}$	Upper-bound steering axis position for 2-DOFW $i$ within $\Delta t$ time (encoders)
	$\dot{s}_{lWi}$	Lower-bound steering axis velocity for 2-DOFW $i$ within $\Delta t$ time (encoders/s)
	$\dot{s}_{uWi}$	Upper bound steering axis velocity for 2-DOFW $i$ within $\Delta t$ time (encoders/s)
25	$t_{mWi}$	Measured translation axis position for 2-DOFW $i$ (encoders)
	$\gamma t_{mWi}$	Measured translation axis position for 2-DOFW $i$ in the previous control cycle (encoders)
30	$\Delta t_{mWi}$	Measured change in translation axis position for 2-DOFW $i$ since the previous control cycle (m)
	$\dot{t}_{dWi}$	Desired translation axis velocity for 2-DOFW $i$ (encoders/s)
	$\dot{t}_{mWi}$	Measured translation axis velocity for 2-DOFW $i$ (encoders/s)
35	$\dot{t}_{\max Wi}$	Maximum translation axis velocity for 2-DOFW $i$ (encoders/s)
	$\ddot{t}_{\max Wi}$	Maximum translation axis acceleration for 2-DOFW $i$ (encoders/s <sup>2</sup> )
40	$\dot{t}_{lWi}$	Lower-bound translation axis velocity for 2-DOFW $i$ within $\Delta t$ time (encoders/s)
	$\dot{t}_{uWi}$	Upper-bound translation axis velocity for 2-DOFW $i$ within $\Delta t$ time (encoders/s)

	$r_{Wi}$	Radius of 2-DOFW $i$ (m)
	$\sigma_{Wi}$	Steering axis encoder pitch for 2-DOFW $i$ (encoders/rad)
5	$\tau_{Wi}$	Translation axis encoder pitch for 2-DOFW $i$ (encoders/rad)
	$c_{Wi}$	Amount of caster for 2-DOFW $i$ (m)
	$\Delta t$	Time elapsed during a control cycle (s)
10	$[x_{eB}, y_{eB}, \psi_{eB}]^T$	Estimated position and orientation of the base coordinate frame (i.e. coordinates that are fixed with respect to the mobile base and move with the mobile base) in fixed world coordinates (i.e. coordinates that are fixed with respect to the floor on which the mobile base moves) (m, m, rad)
15	$C$	Constraint matrix which maps mobile base motion to axis speeds $[2N \times 3]$
	$C_f^\#$	The Force Projection Matrix which is the particular generalized inverse of the $C$ matrix which is used to map the desired motion vector to the actuator control torques $[3 \times 2N]$
20	$C_x^\#$	The Velocity Estimation Matrix which is the particular generalized inverse of the $C$ matrix which is used to map actuator speeds to base speeds $[3 \times 2N]$
	$\Lambda$	The base mass matrix $[3 \times 3]$
	$\mu$	The base centripetal, coriolis, and gravity vector $[3 \times 1]$
25	$\gamma_{dWi}$	Desired steering axis torque for 2-DOFW $i$ (Nm)
	$\gamma_{lWi}$	Lower-bound steering axis torque for 2-DOFW $i$ (Nm)
	$\gamma_{uWi}$	Upper-bound steering axis torque for 2-DOFW $i$ (Nm)
	$\rho_{dWi}$	Desired translation axis torque for 2-DOFW $i$ (Nm)
30	$\rho_{lWi}$	Lower-bound translation axis torque for 2-DOFW $i$ (Nm)
	$\rho_{uWi}$	Upper-bound translation axis torque for 2-DOFW $i$ (Nm)

35 Referring to Figures 1A and 1B, a mobile base constructed according to the present invention includes as many two degrees of freedom wheels (2-DOFWs) as deemed necessary ( $N \geq 2$ , where  $N$  is the number of 2-DOFWs). All of the wheels are mounted on the mobile base which is a rigid platform (depicted in Figure 4.). Each 2-DOFW has two independent axes, one for steering and one for translation.



Figure 1A shows an example of a 2-DOFW with no caster, i.e.  $c_{wi} = 0$ , such that the steering and translation axes intersect. Figure 1B shows an example of a 2-DOFW having a caster offset, i.e.  $c_{wi} \neq 0$ . In other words, the steering axis is offset from the translation axis by  $c_{wi}$ . Each of the 2-DOFWs on the mobile base can have a different amount of caster if necessary.

Referring to Figure 2, a schematic diagram shows the flow of command information which controls the motion of the mobile base. Since each of the  $N$  wheels has two axes (i.e. steering and translation), the mobile base has a total of  $2N$  axes. Controlling the  $2N$  axes of the mobile base are  $2N$  servo amplifiers connected to a set of low-level controllers that perform closed-loop, high servo-rate control of all axis positions. Positional feedback of each axis is provided by an accurate encoding scheme. A supervisory controller interfaced to the low-level controllers coordinates all  $2N$  axes by sending position updates to the low-level controllers at each discrete control cycle. The position updates are calculated by the control algorithm which takes into account the mobile base geometry, motor dynamics, and a 3-DOF input vector sent from a host processor interfaced to the supervisory controller. The 3-DOF input vector completely describes the desired velocity-based motion of the mobile base, which is constrained to move within three DOFs as described previously. A possible input vector, for example, consists of an x-velocity, y-velocity, and rotational velocity with respect to the center of the mobile base, or alternatively, angle, magnitude, and rotational velocity with respect to a random fixed point (i.e. the representation is arbitrary as long as the axes of the input vector are independent.)

The control algorithm is optimal in that it controls the  $2N$  axes such that the mobile base moves as commanded by the input vector as accurately and as quickly as possible within the physical limits of the motors.

For example, if a motor is commanded beyond what it is physically capable of (i.e. it is commanded beyond its "saturation point") while other motors are commanded to within their physical limits, wheel slippage occurs. The control algorithm is able to predict this situation and correct it before it occurs. It accomplishes this by anticipating velocity or torque and saturation points with working models of each motor axis. Thus, the control algorithm simultaneously

minimizes wheel slippage and minimizes the difference between desired motion specified by the input vector and actual mobile base motion (motion error).

## Section 2 - Mapping the Input Vector to the Desired Axis Motion Vector

This determines the axis motion vector (AMV),  $\vec{m}_a$ , which is the motion required at each 2-DOFW and the corresponding motion at each axis such that the mobile base moves according to the commanded input vector (i.e.  $[\dot{x}_d, \dot{y}_d, \dot{\psi}_d]^T$ ). Thus, the mapping is from  $\mathcal{R}^3$  to  $\mathcal{R}^{2N}$ . It is accomplished by first calculating the desired velocity to each wheel attachment point as below. The desired wheel velocity for each wheel is expressed as a 2-vector  $[\dot{x}_{dw_i}, \dot{y}_{dw_i}]^T$  in base coordinates (Figure 4).  
 10  $(\forall i: 1 \leq i \leq N)$  (i.e. for all wheels i):

$$\dot{x}_{dw_i} = \dot{x}_d - y_{wi} \dot{\psi}_d$$

$$\dot{y}_{dw_i} = \dot{y}_d + x_{wi} \dot{\psi}_d$$

where the  $[x_{wi}, y_{wi}]^T$  are the Cartesian coordinates of the wheel attachment point of each 2-DOFW  $i$  to the base in base coordinates (m, m).

15 The steering angle ( $\theta_{mwi}$ ) for each 2-DOFW is measured based on the raw measured encoder value of the steering axis ( $s_{mwi}$ ) and the encoder pitch ( $\sigma_{wi}$ ):

$$\theta_{mwi} = \frac{s_{mwi}}{\sigma_{wi}}$$

and will be used in many of the following calculations.

These desired wheel velocities are then mapped to the two axes of the 2-DOFW. For 2-DOFWs with no caster ( $c_{wi} = 0$ ) the desired steering axis position and desired translation axis velocity  $[s_{dw_i}, \dot{t}_{dw_i}]$  are calculated for each 2-DOFW as follows:

$$\theta_{dw_i} = \arctan2(\dot{y}_{dw_i}, \dot{x}_{dw_i})$$

$$\dot{\mu}_{dw_i} = \sqrt{\dot{x}_{dw_i}^2 + \dot{y}_{dw_i}^2}$$

$$25 \quad s_{dw_i} = \sigma_{wi} \theta_{dw_i}$$

$$\dot{t}_{dw_i} = \frac{\tau_{wi}}{r_{wi}} \dot{\mu}_{dw_i}$$

with

$$\bar{m}_a = [s_{dW1}, \dot{i}_{dW1}, \dots, s_{dWi}, \dot{i}_{dWi}, \dots, s_{dWN}, \dot{i}_{dWN}]^T$$

where

$\theta_{dWi}$  is the desired steering angle for 2-DOFW  $i$ ,

$\sigma_{Wi}$  is the steering axis “encoder pitch” for 2-DOFW  $i$  expressed in encoders per

5 radian, and

$\dot{\mu}_{dWi}$  is the desired velocity magnitude for wheel  $i$

When using a velocity based low level controller for 2-DOFWs with caster  
( $c_{Wi} \neq 0$ ) the desired steering axis velocity and desired translation axis velocity

10  $[\dot{s}_{dWi}, \dot{i}_{dWi}]$  are calculated for each 2-DOFW as below.

$$\dot{s}_{dWi} = \frac{\sigma_{Wi}}{c_{Wi}} (\dot{y}_{dWi} \cos(\theta_{mWi}) - \dot{x}_{dWi} \sin(\theta_{mWi})) - \dot{\psi}$$

$$\dot{i}_{dWi} = \frac{\tau_{Wi}}{r_{Wi}} (\dot{x}_{dWi} \cos(\theta_{mWi}) + \dot{y}_{dWi} \sin(\theta_{mWi}))$$

with

$$\bar{m}_a = [\dot{s}_{dW1}, \dot{i}_{dW1}, \dots, \dot{s}_{dWi}, \dot{i}_{dWi}, \dots, \dot{s}_{dWN}, \dot{i}_{dWN}]^T$$

15 where

$c_{Wi}$  is the amount of caster offset in meters,

$\tau_{Wi}$  is the encoder pitch for 2DOFW  $i$  expressed in encoders per radians, and

$r_{Wi}$  is the radius of the 2DOFW  $i$

When using a torque based low level controller for 2-DOFWs with caster

20 ( $c_{Wi} \neq 0$ ) the desired steering axis torque and desired translation axis torque

$[\gamma_{dWi}, \rho_{dWi}]^T$  are calculated for each 2-DOFW as below. First, gather coefficients of  
the base velocities from the previous equations used in the development of the  
velocity controller into the constraint matrix,  $C$ . The constraint matrix is defined by  
the ideal kinematic relationship:

$$25 \quad \bar{m}_a = C \bar{m}_x$$

where  $\bar{m}_a$  is the motion axis vector and  $\bar{m}_x$  is the actual mobile base motion. The  
constraint matrix for the preferred embodiment is:

$$C = \begin{bmatrix} -\frac{\sigma_{W1}}{c_{W1}} \sin(\theta_{mW1}) & \frac{\sigma_{W1}}{c_{W1}} \cos(\theta_{mW1}) & \frac{\sigma_{W1}}{c_{W1}} (x_{W1} \cos(\theta_{mW1}) + y_{W1} \sin(\theta_{mW1})) - 1 \\ \frac{\tau_{W1}}{r_{W1}} \cos(\theta_{mW1}) & \frac{\tau_{W1}}{r_{W1}} \sin(\theta_{mW1}) & \frac{\tau_{W1}}{r_{W1}} (x_{W1} \sin(\theta_{mW1}) - y_{W1} \cos(\theta_{mW1})) \\ \vdots & \vdots & \vdots \\ -\frac{\sigma_{Wi}}{c_{Wi}} \sin(\theta_{mWi}) & \frac{\sigma_{Wi}}{c_{Wi}} \cos(\theta_{mWi}) & \frac{\sigma_{Wi}}{c_{Wi}} (x_{Wi} \cos(\theta_{mWi}) + y_{Wi} \sin(\theta_{mWi})) - 1 \\ \frac{\tau_{Wi}}{r_{Wi}} \cos(\theta_{mWi}) & \frac{\tau_{Wi}}{r_{Wi}} \sin(\theta_{mWi}) & \frac{\tau_{Wi}}{r_{Wi}} (x_{Wi} \sin(\theta_{mWi}) - y_{Wi} \cos(\theta_{mWi})) \\ \vdots & \vdots & \vdots \\ -\frac{\sigma_{WN}}{c_{WN}} \sin(\theta_{mWN}) & \frac{\sigma_{WN}}{c_{WN}} \cos(\theta_{mWN}) & \frac{\sigma_{WN}}{c_{WN}} (x_{WN} \cos(\theta_{mWN}) + y_{WN} \sin(\theta_{mWN})) - 1 \\ \frac{\tau_{WN}}{r_{WN}} \cos(\theta_{mWN}) & \frac{\tau_{WN}}{r_{WN}} \sin(\theta_{mWN}) & \frac{\tau_{WN}}{r_{WN}} (x_{WN} \sin(\theta_{mWN}) - y_{WN} \cos(\theta_{mWN})) \end{bmatrix}$$

Calculate,  $C_f^\#$ , the force projection matrix, a generalized left inverse of  $C$ .

Any generalized left inverse will work. By judicious choice of a particular  $C_f^\#$  various behaviors can be implemented. An example which is particularly useful is:

$$5 \quad C_f^\# = (C^T C)^{-1} C^T$$

which minimizes, in a least squares way, the axis torques. The axis motion vector containing the desired steering axis torque and desired translation axis torque

$[\gamma_{dWi}, \rho_{dWi}]^T$  is then:

$$\bar{m}_a = C_f^{\#T} \bar{m}_d$$

10 where  $\bar{m}_d$  is the 3-DOF force torque input vector. For control of a base with a known dynamic model, one can dynamically decouple the undesired forces by calculating the axis torques with the following expression:

$$\bar{m}_a = C_f^{\#T} (\Lambda \bar{m}_d + \mu)$$

where  $\bar{m}_d$  is the 3 DOF acceleration input vector and

$$15 \quad \bar{m}_a = [\gamma_{dW1}, \rho_{dW1}, \dots, \gamma_{dWi}, \rho_{dWi}, \dots, \gamma_{dWN}, \rho_{dWN}]^T$$

are the axis torques.

When controlling the base in 3 DOF it is desirable to eliminate or greatly reduce the undesired motion of the vehicle due to the dynamic effects of the various motions of the 2-DOFWs. This can be accomplished by using  $\Lambda$  and  $\mu$  in the above equations as

20 found from the dynamic model of the base. It is well understood by those skilled in

the art that the dynamic model, that is defined by the dynamic equations of a system (base) can be found and written as:

$$F = \Lambda \ddot{x} + \mu$$

where  $F$  is the 3 DOF linear and rotational force on the base,  $\Lambda$  is the mass matrix,

- 5  $\ddot{x}$  is the acceleration of the base in 3DOF, and  $\mu$  is the centripetal, coriolis, and gravity vector. It is possible to control a base with unknown dynamics by using an estimate of the dynamic parameters such as:

$$\Lambda = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mu = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

- 10 but the base will not produce the desired motion as closely as when the values of the dynamic parameters for the actual base are used.

Thus, by concatenating all N wheel motion pairs described above, we create a desired axis motion vector which is 2N in length.

### Section 3 - Calculating the Control Envelopes

- 15 The control envelope for an axis describes the possible motion an axis can perform within a fixed time. For a 2-DOFW  $i$  with no caster ( $c_{wi} = 0$ ) we must calculate the steering axis *position* lower and upper bounds ( $s_{lw_i}$  and  $s_{uw_i}$ ) and the translation axis *velocity* lower and upper bounds ( $\dot{i}_{lw_i}$  and  $\dot{i}_{uw_i}$ ). First, calculating the steering position lower bound, we assume that the axis should move based on a value proportional to the error ( $s_{dw_i} - s_{mw_i}$ ) scaled by gain value ( $k_p$ ) minus a small
- 20 tolerance ( $e_s$ ) or based on the maximum (negative) acceleration possible for the steering axis ( $\ddot{s}_{\max wi}$ ) (the measured position and velocity ( $s_{mw_i}$  and  $\dot{s}_{mw_i}$ ) are required to calculate the position as a result of maximum acceleration), whichever is greater:

$$s_{lw_i} = \max(s_{mw_i} + k_p(s_{dw_i} - s_{mw_i}) - e_s, s_{mw_i} + \dot{s}_{mw_i}\Delta t - \frac{1}{2}\ddot{s}_{\max wi}\Delta t^2)$$

- 25 Calculating the upper bound, we similarly assume the axis should move based on a value proportional to the error scaled by a gain value *plus* a small tolerance or based on the maximum (*positive*) acceleration, whichever is *lesser*.

$$s_{uw_i} = \min(s_{mw_i} + k_p(s_{dw_i} - s_{mw_i}) + e_s, s_{mw_i} + \dot{s}_{mw_i}\Delta t + \frac{1}{2}\ddot{s}_{\max wi}\Delta t^2)$$

Calculating the lower bound of the translation axis velocity, we assume the control envelope is determined by either maximum (negative) acceleration, or by the maximum possible (negative) velocity of the translation axis ( $\dot{i}_{\max W_i}$ ), whichever is greater.

$$5 \quad \dot{i}_{lW_i} = \max(\dot{i}_{mW_i} - \ddot{i}_{\max W_i} \Delta t, -\dot{i}_{\max W_i})$$

Calculating the upper bound, we similarly assume the control envelope is determined by either the maximum (*positive*) acceleration, or by the maximum possible (*positive*) velocity of the translation axis, whichever is *lesser*.

$$10 \quad \dot{i}_{uW_i} = \min(\dot{i}_{mW_i} + \ddot{i}_{\max W_i} \Delta t, \dot{i}_{\max W_i})$$

Once these values have been determined, the desired motion  $[s_{dW_i}, \dot{i}_{dW_i}]$  lies within the control envelope for  $\Delta t$  time duration if and only if

$$s_{dW_i} \geq s_{lW_i} \text{ and } s_{dW_i} \leq s_{mW_i}$$

or

$$15 \quad s_{dW_i} \leq s_{uW_i} \text{ and } s_{dW_i} \geq s_{mW_i}$$

For a 2-DOFW  $i$  with caster ( $c_{W_i} \neq 0$ ), when a velocity based low level controller is used, we must calculate the steering axis velocity lower and upper bounds ( $s_{lW_i}$  and  $s_{uW_i}$ ) and translation axis velocity lower and upper bounds ( $\dot{i}_{lW_i}$  and  $\dot{i}_{uW_i}$ ). Calculating the lower bound of the steering axis velocity, we assume the control envelope is determined by either maximum (negative) acceleration ( $\ddot{s}_{\max W_i}$ ), or by the maximum possible (negative) velocity of the translation axis ( $\dot{s}_{\max W_i}$ ), whichever is greater.

$$25 \quad \dot{s}_{lW_i} = \max(\dot{s}_{mW_i} - \ddot{s}_{\max W_i} \Delta t, -\dot{s}_{\max W_i})$$

Calculating the upper bound, we similarly assume the control envelope is determined by either the maximum (*positive*) acceleration, or by the maximum possible (*positive*) velocity of the steering axis, whichever is *lesser*.

$$30 \quad \dot{s}_{uW_i} = \min(\dot{s}_{mW_i} + \ddot{s}_{\max W_i} \Delta t, \dot{s}_{\max W_i})$$

Calculating the translation velocity lower and upper bounds is the same for 2-DOFWs with caster.

$$\dot{i}_{lWi} = \max(\dot{i}_{mWi} - \ddot{i}_{\max Wi} \Delta t, -\dot{i}_{\max Wi})$$

$$\dot{i}_{uWi} = \min(\dot{i}_{mWi} + \ddot{i}_{\max Wi} \Delta t, \dot{i}_{\max Wi})$$

5

Once these values have been determined, the desired motion  $[\dot{s}_{dWi}, \dot{i}_{dWi}]$  lies within the control envelope for  $\Delta t$  time duration if and only if

$$\dot{s}_{lWi} \leq \dot{s}_{dWi} \leq \dot{s}_{uWi}$$

and

$$10 \quad \dot{i}_{lWi} \leq \dot{i}_{dWi} \leq \dot{i}_{uWi}$$

For a 2-DOFW  $i$  with caster ( $c_{Wi} \neq 0$ ), when a torque based low level controller is used, we must determine if the motion input vector ( $\bar{m}_d$ ) is within the upper bound of the actuator torque ( $\gamma_{uWi}, \rho_{uWi}$ ) and the lower bound of the actuator torque ( $\gamma_{lWi}, \rho_{lWi}$ ):

15

$$\gamma_{lWi} \leq \gamma_{dWi} \leq \gamma_{uWi}$$

$$\rho_{lWi} \leq \rho_{dWi} \leq \rho_{uWi}$$

Thus we can determine for a given increment in time ( $\Delta t$ ) the possible motion for each axis (control envelope) of each 2-DOFW, and for a given axis motion vector, whether it lies within the control envelopes.

20

#### Section 4 - Calculating the Modified Axis Motion Vector

We describe an algorithm that can be applied when the desired axis motion vector mapped from the input vector does not lie within the control envelopes of all axes. Given a desired base motion input vector ( $\bar{m}_d$ ) and a current estimated base motion vector ( $\bar{m}_e$ ) (calculated in Section 5) we can determine a modified motion vector that is closest to the desired input vector while staying within the control envelopes of all axes.

25

When a velocity based low level controller is used, we begin by defining a parametric line  $m_i(\lambda)$  as a function of  $\lambda$ , which varies between 0 and 1:

$$\bar{m}_i(\lambda) = \lambda(\bar{m}_d - \bar{m}_e) + \bar{m}_e$$

$$0 \leq \lambda \leq 1$$

5        Evaluating  $m_i(0)$  results in  $\bar{m}_e$ , which is the current base motion vector. Evaluating  $m_i(1)$  results in  $\bar{m}_d$ , which is the desired input vector. Thus, we define an algorithm that finds a value of  $\lambda$  whose  $m_i(\lambda)$  mapping is closest to the desired input vector *and* lies within the control envelopes when mapped to the axis motion vector. The basic idea is to increment  $\lambda$  by a small amount ( $\delta$ ) until the mapping of  $m_i(\lambda)$  to the axis motion vector lies within the control envelopes. This becomes the modified axis motion vector.

When a torque based low level controller is used, we again begin by defining a parametric line  $m_i(\lambda)$  as a function of  $\lambda$ , which varies between 0 and 1, where now:

15         $\bar{m}_i(\lambda) = \lambda \bar{m}_d$

$$0 \leq \lambda \leq 1$$

Evaluating  $m_i(0)$  results in zero torque command such that the base continues its current base motion vector. Evaluating  $m_i(1)$  results in  $\bar{m}_d$ , which is the desired input vector. Thus, we define an algorithm that finds a value of  $\lambda$  whose  $m_i(\lambda)$  mapping is closest to the desired input vector *and* lies within the control envelopes when mapped to the axis motion vector. The basic idea is to increment  $\lambda$  by a small amount ( $\delta$ ) until the mapping of  $m_i(\lambda)$  to the axis motion vector lies within the control envelopes. This becomes the modified axis motion vector.

25        Using: the appropriate equation for  $m_i(\lambda)$  from above, and with  $0 < \delta \ll 1$  :  
begin procedure

$$\lambda = \delta$$

while  $\bar{m}_i(\lambda)$  mapped to axis motion vector (Section 2) lies within control envelopes of all axes of each 2-DOFW (Section 3) do

30         $\lambda = \lambda + \delta$

end



$$\bar{m}_m = \bar{m}_l(\lambda)$$

return axis motion vector mapping of  $\bar{m}_m$

end

- 5 Note, there are many possible ways to implement the same algorithm. This method is presented because of its simplicity.

### Section 5 - Estimating the Motion of the Mobile Base

- Here, we describe how to estimate the motion of the base during the discrete time interval  $\Delta t$ . First, we calculate the new apparent wheel positions with respect to  
10 the base coordinates. These new coordinates are expressed as  $x'_{wi}$  and  $y'_{wi}$  for wheel i. For wheels with no caster ( $c_{wi} = 0$ ), we begin by calculating the measured steering angle as in Section 1:

$$\theta_{mwi} = \frac{s_{mwi}}{\sigma_{wi}}$$

- We proceed by calculating the measured translation distance ( $\Delta t_{mwi}$ ) since the  
15 beginning of the previous control cycle:

$$\Delta t_{mwi} = (t_{mwi} - \gamma_{mwi})$$

- Here, we introduce  $\gamma_{mwi}$  which is the measured translation axis position at the beginning of the previous control cycle expressed in encoders. The apparent wheel positions are then simply the wheel positions in base coordinates ( $x_{wi}$  and  $y_{wi}$ ) plus  
20 the calculated motion:

$$x'_{wi} = x_{wi} + \Delta t_{mwi} \frac{r_{wi}}{\tau_{wi}} \cos(\theta_{mwi})$$

$$y'_{wi} = y_{wi} + \Delta t_{mwi} \frac{r_{wi}}{\tau_{wi}} \sin(\theta_{mwi})$$

- For wheels with caster ( $c_{wi} \neq 0$ ), we similarly calculate the steering angle,  
25  $\theta_{mwi}$ , and the apparent wheel motion as a result of the translation axis,  $\Delta t_{mwi}$ . However, because of the caster offset, steering motion results in apparent wheel

motion as well. Here we introduce  $s_{mWi}$  which is the measured steering axis position at the beginning of the previous control cycle expressed in encoders:

$$\Delta s_{mWi} = (s_{mWi} - s_{mWi})$$

The apparent wheel motion is then calculated by a rotation with respect to the steer angle:

$$x'_{Wi} = x_{Wi} + \Delta t_{mWi} \frac{r_{Wi}}{\tau_{Wi}} \cos(\theta_{mWi}) - \Delta s_{mWi} \frac{c_{Wi}}{\sigma_{Wi}} \sin(\theta_{mWi})$$

$$y'_{Wi} = y_{Wi} + \Delta s_{mWi} \frac{c_{Wi}}{\sigma_{Wi}} \cos(\theta_{mWi}) + \Delta t_{mWi} \frac{r_{Wi}}{\tau_{Wi}} \sin(\theta_{mWi})$$

We then use the apparent wheel motion to calculate the amount of base rotation  $\Delta\psi_e$  since the last control cycle. We do this by calculating the rotation for each possible wheel pair and averaging the results:

$$M = C(N,2)$$

$$\Delta\psi_e = \frac{1}{M} \sum_{\forall \{j,k\}} \arctan2(y'_{Wk} - y'_{Wj}, x'_{Wk} - x'_{Wj}) - \arctan2(y_{Wk} - y_{Wj}, x_{Wk} - x_{Wj})$$

The above equation is summed over all possible wheel pairs  $j, k$ . That is, there are  $C(N,2)$  ( $N$  choose 2 combinatorial) possible wheel combinations.

Since the position of a mobile base having only two wheels (i.e.  $N = 2$ ) can be estimated with feedback from only three of its four motion axes (the fourth one being redundant), there are many other ways to estimate the motion of the base. The preceding formula can be used for bases having only two wheels, but there will only be one wheel pair to “average.” As the number of wheels used on a mobile base goes up (i.e. as  $N$  increases), the better the above estimation algorithm gets.

We now calculate the change in x and y coordinates ( $\Delta x_e$  and  $\Delta y_e$ ) by evaluating the following equations:

$$\Delta x_e = \frac{1}{N} \sum_{i=1}^N (x'_{Wi} + y_{Wi} \Delta\psi_e)$$

$$\Delta y_e = \frac{1}{N} \sum_{i=1}^N (y'_{wi} - x_{wi} \Delta \psi_e)$$

The estimated base motion vector can be calculated by dividing by the time increment  $\Delta t$  :

$$\bar{m}_e = \frac{1}{\Delta t} [\Delta x_e, \Delta y_e, \Delta \psi_e]^T$$

A similar method to calculate  $\bar{m}_e$ , which directly averages the measured readings with a more compact notation, uses the matrix,  $C$ , introduced above:

$$\bar{m}_e = \frac{1}{\Delta t} C_x^\# [\Delta s_{mW1}, \Delta t_{mW1}, \dots, \Delta s_{mWl}, \Delta t_{mWl}, \dots, \Delta s_{mWN}, \Delta t_{mWN}]^T$$

where  $C_x^\#$ , the velocity estimation matrix, is a generalized left inverse of  $C$ . Any generalized left inverse will work. By judicious choice of a particular  $C_x^\#$  various behaviors can be implemented. An example which is particularly useful is:

$$C_x^\# = (C^T C)^{-1} C^T$$

which minimizes, in a least squares way, the difference in the measured motion of the axis sensors and the motion of axis sensors on an ideal model of the PCV which perfectly obeys the velocity relationship described by  $C$ .

The final step in motion estimation is determining the “summed-up” position of the mobile base in fixed world coordinates. This is accomplished by adding the rotation angle change  $\Delta \psi_e$  to the existing angle estimate ( $\psi_{eB}$ ). Similarly  $x_{eB}$  and  $y_{eB}$  are calculated by adding to the existing estimates the rotated change in x and y coordinates with respect to  $\psi_{eB}$  :

$$\begin{aligned} \psi_{eB} &= \psi_{eB} + \Delta \psi_e \\ x_{eB} &= x_{eB} + \Delta x_e \cos(\psi_{eB}) - \Delta y_e \sin(\psi_{eB}) \\ y_{eB} &= y_{eB} + \Delta y_e \cos(\psi_{eB}) + \Delta x_e \sin(\psi_{eB}) \end{aligned}$$

## Section 6 - Summary

The present invention, as described above, provides a method and apparatus for controlling the motion of a mobile base with increased accuracy and maneuverability. In its preferred embodiment, the present invention is used on a mobile robot base having three wheels each with a predetermined amount of caster. The mobile robot is controlled by an off-board host processor (as shown in Figure 2.)

The host processor sends command signals and receives motion feedback from an onboard supervisory controller by radio, cable, infrared, or similar type of link. The motion control signals are mapped to axis control signals by the supervisory controller and sent to six low-level controllers. Each of the six low level controllers corresponds to either a steering axis or translation axis for one of the three wheels. Each of the low level controllers in turn sends an axis control signal to an associated servo amplifier, which provides the proper voltage and current to drive the respective axis motor. Each of the six motors has an encoder, which provides motor position feedback to both the low-level controller and the supervisory controller.

The host processor and supervisory controller are preferably microprocessors that are commonly used for embedded control. Propriety software code is written preferably in C programming language to implement the inventive control method on the microprocessors and low level controllers.

The inventive method and apparatus can be utilized with other configurations (not shown), such as on drive systems for forklifts or automated guided vehicles (AGV's.) Also, the mobile base described above can be inverted with the positions of the mobile base and the surface it rolls on transposed. In other words, two degree of freedom wheels can be mounted pointed upward on a stationary base, and can translate and rotate a horizontal surface resting on the wheels. In another possible application (not shown), multiple bases, each having single or multiple wheels, can be pivotably linked together in a snake-fashion to form a non-rigid base which is controlled by the inventive method.

The above descriptions and drawings are for illustrative purposes only, and are not exhaustive of possible alternate embodiments of the invention. It is to be understood that the present invention is not limited to the sole embodiments described above and illustrated herein, but encompasses any and all variations falling within the scope of the appended claims.

What is claimed is:

1. A method of calculating a desired axis motion of a wheeled base from an overall desired motion of the wheeled base, the wheeled base having at least two wheels, each wheel having two axis which define two degrees of freedom for that wheel, each wheel having an attachment point on the base, the method comprising the steps of:

receiving an inputted motion vector for an overall desired motion of the wheeled base;

10 calculating from the inputted vector a desired two dimensional motion vector for each of the attachment points;

mapping the motion vector of one of the attachment points to an axis motion for each of the two axes associated with the one attachment point, so that driving the two axes of the associated wheel will result in the desired base motion of the one attachment point; and

15 repeating the previous step for each of the attachment points.

2. A method of calculating a desired axis motion as recited in claim 1, wherein the step of calculating the two dimensional motion vector for each of the attachment points is characterized by the equations:

$$\dot{x}_{dwi} = \dot{x}_d - y_{wi}\dot{\psi}_d$$

$$\dot{y}_{dwi} = \dot{y}_d + x_{wi}\dot{\psi}_d$$

wherein

$\dot{x}_{dwi}$  is the desired wheel attachment point velocity in the x-direction of the base coordinates for each wheel  $i$  in meters per second,

$\dot{x}_d$  is the desired base velocity in the x-direction of the base coordinates in meters per second,

$\dot{\psi}_d$  is the desired rotational velocity in base coordinates in radians per second,

$\dot{y}_{dwi}$  is the desired wheel attachment point velocity in the y direction of the base coordinates for each wheel  $i$  in meters per second,

$\dot{y}_d$  is the desired base velocity in the x-direction of the base coordinates in meters per second,

$x_{wi}$  is the x-component of the wheel attachment point of wheel  $i$  in base coordinates in meters, and

5  $y_{wi}$  is the y-component of the wheel attachment point of wheel  $i$  in base coordinates in meters.

3. A method of calculating a desired axis motion as recited in claim 1, wherein all of the wheels of the base have a caster that is equal to zero and the steps of mapping the motion vectors of the attachment points to axis motions is characterized

10 by the equations:

$$\theta_{dwi} = \arctan2(\dot{y}_{dwi}, \dot{x}_{dwi})$$

$$\dot{\mu}_{dwi} = \sqrt{\dot{x}_{dwi}^2 + \dot{y}_{dwi}^2}$$

$$s_{dwi} = \sigma_{wi} \theta_{dwi}$$

$$\dot{t}_{dwi} = \frac{\tau_{wi}}{r_{wi}} \dot{\mu}_{dwi}$$

15 wherein

$\theta_{dwi}$  is the desired steering angle for 2-DOFW  $i$ , in radians,

$\dot{y}_{dwi}$  is the desired wheel attachment point velocity in the y direction of the base coordinates for each wheel  $i$  in meters per second,

$\dot{x}_{dwi}$  is the desired wheel attachment point velocity in the x-direction of the base

20 coordinates for each wheel  $i$  in meters per second,

$\dot{\mu}_{dwi}$  is the desired velocity magnitude for wheel  $i$  in meters per second,

$s_{dwi}$  is the desired steering angle with respect to base coordinates of wheel  $i$  in encoders,

$\sigma_{wi}$  is the steering axis “encoder pitch” for the steering axis of 2-DOFW  $i$  expressed in  
25 encoders per radian,

$\dot{t}_{dwi}$  is the desired velocity of the translation axis for wheel  $i$  in encoders per second,

$\tau_{wi}$  is the translation axis “encoder pitch” for wheel  $i$  in encoders per radian, and

$r_{wi}$  is the wheel radius of wheel  $i$  in meters per radian.

4. A method of calculating a desired axis motion as recited in claim 1, wherein all of the wheels of the base have a caster that is not equal to zero and the steps of mapping the motion vectors of the attachment points to axis motions is

5 characterized by the equations:

$$\theta_{mWi} = \frac{s_{mWi}}{\sigma_{Wi}} ;$$

$$\dot{s}_{dWi} = \frac{\sigma_{Wi}}{c_{Wi}} (\dot{y}_{dWi} \cos(\theta_{mWi}) - \dot{x}_{dWi} \sin(\theta_{mWi})) - \dot{\psi}_d ; \text{ and}$$

$$\dot{i}_{dWi} = \frac{\tau_{Wi}}{r_{Wi}} (\dot{x}_{dWi} \cos(\theta_{mWi}) + \dot{y}_{dWi} \sin(\theta_{mWi})),$$

wherein

10  $\theta_{mWi}$  is the measured angle of the steering axis of wheel  $i$  with respect to base coordinates in radians,

$s_{mWi}$  is the measured angle of wheel  $i$  with respect to base coordinates in encoders,

$\sigma_{Wi}$  is the steering axis “encoder pitch” for the steering axis of 2-DOFW- $i$  expressed in encoders per radian,

15  $\dot{s}_{dWi}$  is the desired steering axis velocity of wheel  $i$  in encoders per second,

$c_{Wi}$  is the amount of caster offset in meters per radian,

$\dot{y}_{dWi}$  is the desired wheel attachment point velocity in the y direction of the base coordinate for each wheel  $i$  in meters per second,

$\dot{x}_{dWi}$  is the desired wheel attachment point velocity in the x-direction of the base

20 coordinates for each wheel,  $i$  in meters per second,

$\dot{\psi}_d$  is the desired base rotation velocity in radians per second,

$\dot{i}_{dWi}$  is the desired velocity of the translation axis for wheel  $i$  in encoders per second,

$\tau_{Wi}$  is the encoder pitch for 2DOFW  $i$  expressed in encoders per radians, and

$r_{Wi}$  is the radius of the 2DOFW  $i$  in meters per radian.

25

5. A method of providing a control envelope for axes on a wheeled base, the wheeled base having at least two wheels, each wheel having two axes which

define two degrees of freedom for that wheel, the control envelope ensuring that none of the wheel axes are commanded to move in a manner that they are not capable, the method comprising the steps of:

- 5       defining for each of the axes of each of the wheels a maximum possible range of motion that will be permitted during an allotted control period;
- receiving an inputted motion vector for an overall desired motion of the wheeled base;
- calculating from the inputted vector a desired axis motion for each of the axes of each of the wheels, so that driving all of the axes will result in the overall desired motion of the base;
- determining if any of the desired axis motions are not within the permitted range of motion previously defined for that axis; and
- modifying as little as possible the overall desired motion of the base if required by the previous step such that all of the corresponding desired axis motions are within the permitted ranges of axis motion.

6.       A method of estimating rotation of a wheeled base relative to a surface during a discrete time interval  $\Delta t$ , the base having at least two wheels each pivotably and rotatably mounted thereon for contacting the surface and driving the base and surface relative to one another, the method comprising the steps of:
  - 20       calculating an apparent motion for each of the wheels during the time interval  $\Delta t$ ;
  - calculating a rotation of the base for each possible pair of the wheels based on the apparent motion calculated for each of the wheels; and
  - 25       averaging the results of the previous step.

7.       A method of estimating rotation of a wheeled base as recited in claim 6, wherein the step of calculating an apparent motion of each wheel is represented by the equations:

$$x'_{wi} = x_{wi} + \Delta t_{mwi} \cos(\theta_{mwi}) - \Delta s_{mwi} \sin(\theta_{mwi}) ; \text{ and}$$

$$y'_{wi} = y_{wi} + \Delta s_{mwi} \cos(\theta_{mwi}) + \Delta t_{mwi} \sin(\theta_{mwi}) ,$$



wherein

$x'_{wi}$  is the x-component of the attachment point of wheel  $i$  after base motion during one control cycle in meters,

$x_{wi}$  is the x-component of the wheel attachment point of wheel  $i$  in base coordinates in meters,

$\Delta t_{mwi}$  is the measured motion of the translation axis of wheel  $i$  during one control cycle expressed in meters,

$\theta_{mwi}$  is the measured angle of the steering axis of wheel  $i$  with respect to base coordinates in radians,

10  $\Delta s_{mwi}$  is the measured motion of the steering axis of wheel  $i$  during one control cycle expressed in meters,

$y'_{wi}$  is the y-component of the attachment point of wheel  $i$  after base motion during one control cycle in meters, and

15  $y_{wi}$  is the y-component of the wheel attachment point of wheel  $i$  in base coordinates in meters.

8. A method of estimating rotation of a wheeled base as recited in claim 7, wherein each wheel has a steering axis and a rolling or translation axis, the two axis for each of the wheels being offset from one another such that the caster does not equal zero.

9. A method of estimating rotation of a wheeled base as recited in claim 7, wherein each wheel has a steering axis and a rolling or translation axis, the two axis for each of the wheels intersecting such that the caster equals zero.

25 10. A method of estimating rotation of a wheeled base as recited in claim 6, wherein the steps of calculating a rotation of the base for each possible pair of the wheels and averaging the results are represented by the equation:

$$\Delta \psi_e = \frac{1}{M} \sum_{\forall \{j,k\}} \arctan2(y'_{wk} - y'_{wj}, x'_{wk} - x'_{wj}) - \arctan2(y_{wk} - y_{wj}, x_{wk} - x_{wj}),$$

30 wherein

$\Delta\psi_e$  is the calculated rotation change in radians,

m is the number of unique pair combinations of N wheels,

j and k are index variables of a possible wheel pair combination,

$y'_{wk}$  is the y-component of the attachment point of wheel k after base motion during  
5 one control cycle in meters,

$y'_{wj}$  is the y-component of the attachment point of wheel j after base motion during  
one control cycle in meters,

$x'_{wk}$  is the x-component of the attachment point of wheel k after base motion during  
one control cycle in meters,

10  $x'_{wj}$  is the x-component of the attachment point of wheel j after base motion during  
one control cycle in meters,

$y_{wk}$  is the y-component of the wheel attachment point of wheel k in base coordinates  
in meters,

$y_{wj}$  is the y-component of the wheel attachment point of wheel j in base coordinates  
15 in meters,

$x_{wk}$  is the x-component of the wheel attachment point of wheel k in base coordinates  
in meters, and

$x_{wj}$  is the x-component of the wheel attachment point of wheel j in base coordinates in  
meters.

20

11. A method of calculating position and orientation of a wheeled base  
relative to a world coordinate system fixed to a surface, the base having a base  
coordinate system fixed thereto and at least two wheels each pivotably and rotatably  
mounted to the base at an attachment point for contacting the surface and driving the  
25 base and surface relative to one another, the method comprising the steps of:

calculating a steering angle for each of the wheels;

calculating a measured translation distance for each of the wheels  
traveled over a time  $\Delta t$  elapsed during a control cycle;

30 calculating an apparent motion for each of the wheels based on the  
steering angle and measured translation calculated;

calculating an apparent position in base coordinates for each of the wheels by adding the calculated motion to the wheel attachment point at the beginning of the control cycle;

calculating a rotation of the base for each possible pair of the wheels based on the apparent motion calculated for each of the wheels;

calculating a total change in base rotation by averaging the results of the previous step;

calculating a total change in translation coordinates by averaging the apparent motion of each of the wheels, taking into account the base rotation calculated in the previous step since the beginning of the control cycle;

calculating a summed-up position and orientation of the mobile base in the fixed world coordinates by adding the total change in base angle to an existing angle estimate and adding the total change in translation coordinates to an existing estimate of translation coordinates.

12. A method of calculating position and orientation of a wheeled base as recited in claim 11, wherein the step of calculating the total change in base rotation is characterized by the equation:

$$\Delta\psi_e = \frac{1}{M} \sum_{\forall \{j,k\}} \arctan2(y'_{wk} - y'_{wj}, x'_{wk} - x'_{wj}) - \arctan2(y_{wk} - y_{wj}, x_{wk} - x_{wj}),$$

wherein

$\Delta\psi_e$  is the calculated rotation change in radians,

m is the number of unique pair combinations of N wheels,

j and k are index variables of a possible wheel pair combination,

$y'_{wk}$  is the y-component of the attachment point of wheel k after base motion during one control cycle in meters,

$y'_{wj}$  is the y-component of the attachment point of wheel j after base motion during one control cycle in meters,

$x'_{wk}$  is the x-component of the attachment point of wheel k after base motion during one control cycle in meters,

$x'_{wj}$  is the x-component of the attachment point of wheel j after base motion during one control cycle in meters,

$y_{wk}$  is the y-component of the wheel attachment point of wheel k in base coordinates in meters,

5  $y_{wj}$  is the y-component of the wheel attachment point of wheel j in base coordinates in meters,

$x_{wk}$  is the x-component of the wheel attachment point of wheel k in base coordinated in meters, and

$x_{wj}$  is the x-component of the wheel attachment point of wheel j in base coordinated in meters.

13. A method of calculating position and orientation of a wheeled base as recited in claim 11, wherein the step of calculating the total change in translation coordinates is characterized by the equations:

15 
$$\Delta x_e = \frac{1}{N} \sum_{i=1}^N (x'_{wi} + y_{wi} \Delta \psi_e)$$
$$\Delta y_e = \frac{1}{N} \sum_{i=1}^N (y'_{wi} - x_{wi} \Delta \psi_e)$$

wherein

$\Delta x_e$  is the estimated change of the base location in the x-direction in base coordinates since the previous control cycle in meters,

20 N is the number of wheels,

i is an index variable,

$x'_{wi}$  is the x-component of the attachment point of wheel i after base motion during one control cycle in meters,

$x_{wi}$  is the x-component of the wheel attachment point of wheel i in base coordinates in meters,

25  $y_{wi}$  is the y-component of the wheel attachment point of wheel i in base coordinate in meters,

$\Delta\psi_e$  is the estimated rotational change of the base location in base coordinates since the previous control cycle in meters,

$\Delta y_e$  is the estimated change of the base location in the y-direction in base coordinates since the previous control cycle in meters, and

- 5  $y'_{wi}$  is the y-component of the attachment point of wheel  $i$  after base motion during one control cycle in meters.

14. A method of calculating position and orientation of a wheeled base as recited in claim 11, wherein the step of calculating a summed-up position and  
10 orientation of the mobile base is characterized by the equations:

$$\psi_{eB} = \psi_{eB} + \Delta\psi_e,$$

$$x_{eB} = x_{eB} + \Delta x_e \cos(\psi_{eB}) - \Delta y_e \sin(\psi_{eB}), \text{ and}$$

$$y_{eB} = y_{eB} + \Delta y_e \cos(\psi_{eB}) + \Delta x_e \sin(\psi_{eB})$$

wherein

- 15  $\psi_{eB}$  is the estimated rotational position of the base coordinate frame origin with respect to a fixed world coordinate frame,  
 $\Delta\psi_e$  is the estimated rotational change of the base location in base coordinates since the previous control cycle in meters,  
 $x_{eB}$  is the estimated position of the base coordinate frame origin in the x-direction of a  
20 fixed world coordinate frame,  
 $\Delta x_e$  is the estimated change of the base location in the x-direction in base coordinates since the previous control cycle in meters,  
 $\Delta y_e$  is the estimated change of the base location in the y-direction in base coordinates since the previous control cycle in meters, and  
25  $y_{eB}$  is the estimated position of the base coordinate frame origin in the y-direction of a fixed world coordinate frame.

15. A method for controlling the motion of a wheeled base with respect to a surface, the base having a base coordinate system fixed thereto and at least two  
30 wheels each pivotably and rotatably mounted to the base at an attachment point for

contacting the surface and driving the base and surface relative to one another, each wheel having a plurality of axes, the method comprising the steps of:

reading an input vector from a host processor;

mapping the input vector to a desired axis motion vector for each of the

5 axes;

sending the desired axis motion vector to an axis controller for each of the axes;

estimating a motion of the base traveled during a discrete time interval  $\Delta t$ ;

10 calculating a position and an orientation of the base in a set of world coordinates.

16. A method for controlling the motion of a wheeled base as recited in claim 15, wherein the steps of estimating a motion of the base and calculating a position and orientation of the base further comprise the steps of:

15 calculating a steering angle for each of the wheels;

calculating a measured translation distance for each of the wheels traveled over a time  $\Delta t$  elapsed during a control cycle;

20 calculating an apparent motion for each of the wheels based on the steering angle and measured translation calculated;

calculating an apparent position in base coordinates for each of the wheels by adding the calculated motion to the wheel attachment point at the beginning of the control cycle

25 calculating a rotation of the base for each possible pair of the wheels based on the apparent motion calculated for each of the wheels;

calculating a total change in base rotation by averaging the results of the previous step;

30 calculating a total change in translation coordinates by averaging the apparent motion of each of the wheels, taking into account the base rotation calculated in the previous step since the beginning of the control cycle;

calculating a summed-up position and orientation of the mobile base in the fixed world coordinates by adding the total change in base angle to an

existing angle estimate and adding a rotated total change in translation coordinates to an existing estimate of translation coordinates.

17. A method for controlling the motion of a wheeled base as recited in claim 16, wherein the step of calculating the total change in base rotation is characterized by the equation:

$$\Delta\psi_e = \frac{1}{M} \sum_{\forall \{j,k\}} \arctan2(y'_{wk} - y'_{wj}, x'_{wk} - x'_{wj}) - \arctan2(y_{wk} - y_{wj}, x_{wk} - x_{wj}),$$

wherein

$\Delta\psi_e$  is the calculated rotation change in radians,

- 10 m is the number of unique pair combinations of N wheels,

j and k are index variables of a possible wheel pair combination,

$y'_{wk}$  is the y-component of the attachment point of wheel k after base motion during one control cycle in meters,

- 15  $y'_{wj}$  is the y-component of the attachment point of wheel j after base motion during one control cycle in meters,

$x'_{wk}$  is the x-component of the attachment point of wheel k after base motion during one control cycle in meters,

$x'_{wj}$  is the x-component of the attachment point of wheel j after base motion during one control cycle in meters,

- 20  $y_{wk}$  is the y-component of the wheel attachment point of wheel k in base coordinates in meters,

$y_{wj}$  is the y-component of the wheel attachment point of wheel j in base coordinates in meters,

- 25  $x_{wk}$  is the x-component of the wheel attachment point of wheel k in base coordinates in meters, and

$x_{wj}$  is the x-component of the wheel attachment point of wheel j in base coordinates in meters.

18. A method for controlling the motion of a wheeled base as recited in claim 16, wherein the step of calculating the total change in translation coordinates is characterized by the equations:

$$\Delta x_e = \frac{1}{N} \sum_{i=1}^N (x'_{wi} + y_{wi} \Delta \psi_e); \text{ and}$$

$$\Delta y_e = \frac{1}{N} \sum_{i=1}^N (y'_{wi} - x_{wi} \Delta \psi_e),$$

wherein

$\Delta x_e$  is the estimated change of the base location in the x-direction in base coordinates since the previous control cycle in meters,

10 N is the number of wheels,

i is an index variable,

$x'_{wi}$  is the x-component of the attachment point of wheel  $i$  after base motion during one control cycle in meters,

15  $x_{wi}$  is the x-component of the wheel attachment point of wheel  $i$  in base coordinates in meters,

$y_{wi}$  is the y-component of the wheel attachment point of wheel  $i$  in base coordinate in meters,

$\Delta \psi_e$  is the estimated rotational change of the base location in base coordinates since the previous control cycle in meters,

20  $\Delta y_e$  is the estimated change of the base location in the y-direction in base coordinates since the previous control cycle in meters, and

$y'_{wi}$  is the y-component of the attachment point of wheel  $i$  after base motion during one control cycle in meters.

25 19. A method for controlling the motion of a wheeled base as recited in claim 16, wherein the step of calculating a summed-up position and orientation of the mobile base is characterized by the equations:

$$\psi_{eB} = \psi_{eB} + \Delta \psi_e,$$

$$x_{eB} = x_{eB} + \Delta x_e \cos(\psi_{eB}) - \Delta y_e \sin(\psi_{eB}), \text{ and}$$



$$y_{eB} = y_{eB} + \Delta y_e \cos(\psi_{eB}) + \Delta x_e \sin(\psi_{eB})$$

wherein

$\psi_{eB}$  is the estimated rotational position of the base coordinate frame origin with respect to a fixed world coordinate frame,

5  $\Delta\psi_e$  is the estimated rotational change of the base location in base coordinates since the previous control cycle in meters,

$x_{eB}$  is the estimated position of the base coordinate frame origin in the x-direction of a fixed world coordinate frame,

10  $\Delta x_e$  is the estimated change of the base location in the x-direction in base coordinates since the previous control cycle in meters,

$\Delta y_e$  is the estimated change of the base location in the y-direction in base coordinates since the previous control cycle in meters, and

$y_{eB}$  is the estimated position of the base coordinate frame origin in the y-direction of a fixed world coordinate frame.

15

20. A method for controlling the motion of a wheeled base with respect to a surface, the base having a base coordinate system fixed thereto and at least two wheels each pivotably and rotatably mounted to the base for contacting the surface and driving the base and surface relative to one another, each wheel having a plurality of axes, the method comprising the steps of:

20

reading an input vector from a host processor;

mapping the input vector to a desired axis motion vector for each of the axes;

calculating a control envelope for each of the axes;

25

determining whether the desired axis motion vector lies within the control envelope for each of the axes;

calculating a modified axis motion vector for each of the axes in which the desired axis motion vector does not lie within the control envelope;

sending the desired axis motion vector or modified axis motion vector

30

to an axis controller for each of the axes;

estimating a motion of the base traveled during a discrete time interval  $\Delta t$ ;

calculating a position and an orientation of the base in a set of world coordinates; and

5 repeating the previous steps continuously until commanded to stop.

21. A mobile base movable relative to a surface, the base comprising:  
at least two wheels pivotably and rotatably mounted to the base, each wheel  
having a steering axis and a rotation axis;

10 drive means for rotating the wheels along the surface;

steering means for pivoting the wheels with respect to the surface; and

controller means for controlling the motion of the base, wherein the  
controller means includes means for reading an input vector from a host

15 processor, mapping the input vector to a desired axis motion vector for each of  
the axes, calculating a control envelope for each of the axes, determining

whether the axis motion vector lies within the control envelope for each of the  
axes, calculating a modified axis motion vector when the axis motion vector  
does not lie within the control envelope, sending the axis motion vector or  
modified axis motion vector to an axis controller for each of the axes,

20 estimating a motion of the base traveled during a discrete time interval  $\Delta t$ ,  
calculating a position and an orientation of the base in a set of world  
coordinates, and repeating the previous steps continuously until commanded to  
stop.

25 22. A mobile base movable relative to a surface, the base  
comprising:

at least three wheels pivotably and rotatably mounted to the base, each wheel  
having a steering axis and a rotation axis;

drive means for rotating the wheels along the surface;

30 first encoder means for sensing the rotation of each of the wheels and  
outputting a signal for each wheel in response thereto;

steering means for pivoting the wheels with respect to the surface;  
second encoder means for sensing the pivoting of each of the wheels  
and outputting a signal for each wheel in response thereto; and  
a processor for estimating a rotation of the base relative to the surface  
during a discrete time interval  $\Delta t$  by receiving the output signals from the first  
and second encoder means, calculating from the signals an apparent motion for  
each of the wheels during the time interval  $\Delta t$ , calculating from the apparent  
motions a rotation of the base during the time interval  $\Delta t$  for each possible pair  
of wheels, and averaging the results of the calculated rotations.

23. A mobile base movable relative to a surface, the base  
comprising:

at least two wheels pivotably and rotatably mounted to the base, each wheel  
having a steering axis and a rotation axis;

drive means for rotating the wheels along the surface;  
steering means for pivoting the wheels with respect to the surface;  
a controller for supplying power to the drive means and the steering  
means; and

a processor for sending command signals to the controller and for  
calculating a control envelope to ensure that commands are not sent to the  
controller that cannot be executed by the drive means or the steering means,  
the processor calculating the control envelope by receiving an input motion  
signal indicating an overall desired motion of the base, calculating from the  
input signal a desired axis motion for each of the wheel axes that will achieve  
the overall desired motion of the base, determining if each of the desired axis  
motions is within an associated range of motion predefined for each of the  
axes, and, if required, modifying as little as possible the overall desired motion  
of the base until all of the corresponding desired axis motions are within the  
associated predefined ranges of axis motion.

24. A method of providing a control envelope for axes on a wheeled base,  
the wheeled base having at least two wheels, each wheel having two axes which

define two degrees of freedom for that wheel, the control envelope ensuring that none of the wheel axes are commanded to move in a manner that they are not capable, the method comprising the steps of:

- defining for each of the axes of each of the wheels a maximum permitted  
5 range of torque;- receiving an inputted force vector for an overall desired resultant force of the wheeled base;- calculating from the inputted vector a desired axis torque for each of the axes of each of the wheels, so that driving all of the axes will result in the overall desired  
10 force of the base;- determining if any of the desired axis torques are not within the permitted range of torque previously defined for that axis; and  
modifying as little as possible the overall desired force input vector of the base if required by the previous step such that all of the corresponding desired axis torques  
15 are within the permitted ranges of axis torques.

25. A method of calculating position and orientation of a wheeled base as recited in claim 11, wherein the steps of calculating the total change in rotation and translation coordinates are characterized by the equation:

20 
$$\bar{m}_e = \frac{1}{\Delta t} C_x^\# [\Delta s_{mW1}, \Delta t_{mW1}, \dots, \Delta s_{mWi}, \Delta t_{mWi}, \dots, \Delta s_{mWN}, \Delta t_{mWN}]^T$$

wherein

$\bar{m}_e$  is the estimated mobile base motion in base coordinates.

$\Delta t$  is the time elapsed during a control cycle (s),

25  $C_x^\#$  is the velocity estimation matrix, a generalized left inverse of the constraint matrix,

$\Delta s_{mWi}$  is the measured motion of the steering axis of wheel i during one control cycle,

$\Delta t_{mWi}$  is the measured motion of the translation axis of wheel i during one  
30 control cycle, and

N is the number of wheels mounted to the base.

26. A mobile base movable relative to a surface, the base comprising:  
a main housing;

- 5 at least two wheels pivotably and rotatably mounted to the housing , each  
wheel having a steering axis and a rotation axis with the steering and rotation axes  
nonintersecting and offset by a known caster distance;  
drive means for rotating the wheels to roll along the surface;  
steering means for pivoting the wheels and changing their heading with  
10 respect to the surface; and  
controller means for  
reading an input vector from a host processor, wherein the input vector is a  
three dimensional force torque vector,  
reading the steering axis headings,  
15 calculating a desired torque for each steering and rotation axis such that at any  
given time, the calculated resultant forces on the base reflect the input vector,  
regardless of the positions of the steering and rotation axes, and  
commanding the calculated torque to each steering and rotation axis.

20

27. An apparatus as described in claim 26 wherein the controller means  
includes a dynamic model such that the controller means compensates for undesired  
motion of the base due to various motions of the wheels.

- 25 28. An apparatus as described in claim 26 wherein the calculated torque of  
each steering and rotation axis is computed from the input vector and a generalized  
inverse of a constraint matrix,  $C$  , the constraint matrix being defined by the following  
kinematic relationship:

$$\bar{m}_a = C\bar{m}_x$$

- 30 where  $\bar{m}_a$  represents a motion axis vector and  $\bar{m}_x$  represents actual mobile  
base motion.

29. An apparatus as described in claim 28 wherein the generalized inverse of  $C$  is chosen such that a sum of squares of the axis torques is minimized.

30. An apparatus as described in claim 28 wherein the generalized inverse of  $C$  is chosen such that a sum of squares of wheel contact forces is minimized.

5 31. An apparatus as described in claim 28 wherein there is an instantaneous power for each of the steering and rotation axes and wherein the generalized inverse of  $C$  is chosen such that a sum of the instantaneous powers of all of the axes is minimized.

## Abstract of the Disclosure

A mobile robot base movable relative to a surface, comprising: at least two wheels pivotably and rotatably mounted to the base, each wheel having a steering axis and a rotation axis; drive means for rotating the wheels along the surface; steering  
5 means for pivoting the wheels with respect to the surface; and controller means for controlling the motion of the base, wherein the controller means includes means for reading an input motion vector from a host processor, mapping the input vector to a desired axis motion vector for each of the axes (Fig. 3), calculating a control envelope for each of the axes, determining whether the axis motion vector lies within the  
10 control envelope for each of the axes (Fig. 3), calculating a modified axis motion vector when the axis motion vector does not lie within the control envelope (Fig. 3), sending the axis motion vector or modified axis motion vector to an axis controller for each of the axes (Fig. 3), estimating a motion of the base traveled during a discrete time interval  $\Delta t$  (Fig. 3), calculating a position and an orientation of the base in a  
15 set of world coordinates (Fig. 3), and repeating the previous steps continuously until commanded to stop (Fig. 3). Other aspects include a holonomic base capable of instantaneous movement in any direction without reconfiguring the wheel headings, regardless of the configuration of the wheels. Also, a method of using forces and torques to control the base, and to compensate for undesired motions of the base due  
20 to dynamic effects of the wheel motions is disclosed.

Applicant or Patentee: James C. Slater et al.  
Appn. or Patent No.: New  
Filed or Issued: Herewith  
For: METHOD AND APPARATUS FOR MOBILE ROBOT MOTION CONTROL

Attorney's  
Docket No.: NOMA-110

## VERIFIED STATEMENT (DECLARATION) CLAIMING SMALL ENTITY STATUS (37 CFR 1.9(f) and 1.27(c)) - SMALL BUSINESS CONCERN

I hereby declare that I am

- ☐ the owner of the small business concern identified below:  
☒ an official of the small business concern empowered to act on behalf of the concern identified below:

NAME OF CONCERN Nomadic Technologies

ADDRESS OF CONCERN 2133 Leghorn Street, Mt. View, California 94043-1605

I hereby declare that the above identified small business concern qualifies as a small business concern as defined in 13 CFR 121.3-18, and reproduced in 37 CFR 1.9(d), for purposes of paying reduced fees under section 41(a) and (b) of Title 35, United States Code, in that the number of employees of the concern, including those of its affiliates, does not exceed 500 persons. For purposes of this statement, (1) the number of employees of the business concern is the average over the previous fiscal year of the concern of the persons employed on a full-time, part-time or temporary basis during each of the pay periods of the fiscal year, and (2) concerns are affiliates of each other when either, directly or indirectly, one concern controls or has the power to control the other, or a third party or parties controls or has the power to control both.

I hereby declare that rights under contract or law have been conveyed to and remain with the small business concern identified above with regard to the invention, entitled METHOD AND APPARATUS FOR MOBILE ROBOT MOTION CONTROL by inventor(s) James C. Slater et al. described in

- ☒ the specification filed herewith with title as listed above.  
☐ application no. , filed .  
☐ patent no. , issued .

If the rights held by the above identified small business concern are not exclusive, each individual, concern or organization having rights to the invention is listed below\* and no rights to the invention are held by any person, other than the inventor, who could not qualify as an independent inventor under 37 CFR 1.9(c) if that person made the invention, or by any concern which would not qualify as a small business concern under 37 CFR 1.9(d) or a nonprofit organization under 37 CFR 1.9(e).

\*NOTE: Separate verified statements are required from each named person, concern or organization having rights to the invention averring to their status as small entities. (37 CFR 1.27).

NAME \_\_\_\_\_

ADDRESS \_\_\_\_\_

☐ Individual ☐ Small Business Concern ☐ Nonprofit Organization

NAME \_\_\_\_\_

ADDRESS \_\_\_\_\_

☐ Individual ☐ Small Business Concern ☐ Nonprofit Organization

I acknowledge the duty to file, in this application or patent, notification of any change in status resulting in loss of entitlement to small entity status prior to paying, or at the time or paying, the earliest of the issue fee or any maintenance fee due after the date on which status as a small entity is no longer appropriate. (37 CFR 1.28(b))

I hereby declare that all statements made herein of my own knowledge are true and that all statements made on information and belief are believed to be true; and further that these statements were made with the knowledge that willful false statements and the like so made are punishable by fine or imprisonment, or both, under Section 1001 of Title 18 of the United States Code, and that such willful false statements may jeopardize the validity of the application, any patent issuing thereon, or any patent to which this verified statement is directed.

NAME OF PERSON SIGNING \_\_\_\_\_

TITLE OF PERSON OTHER THAN OWNER James C. Slater

ADDRESS OF PERSON SIGNING Nomadic Technologies, 2133 Leghorn Street

SIGNATURE: \_\_\_\_\_

DATE: \_\_\_\_\_



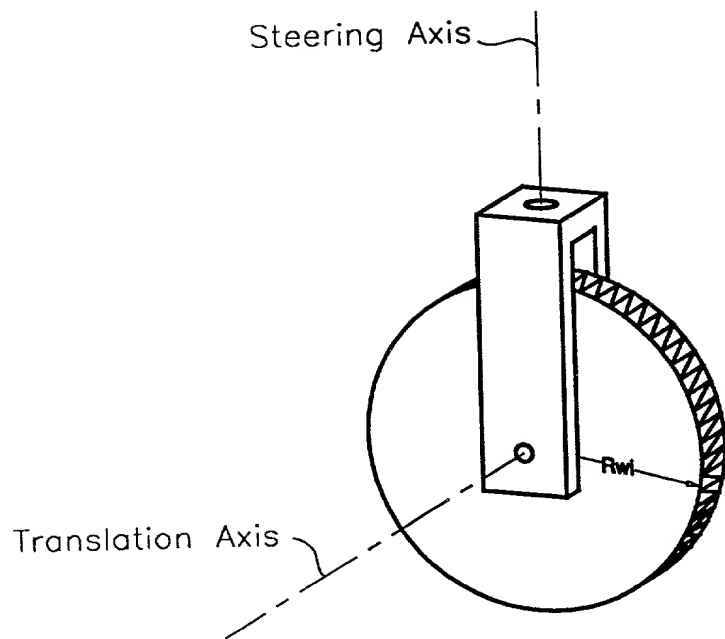


Fig. 1A

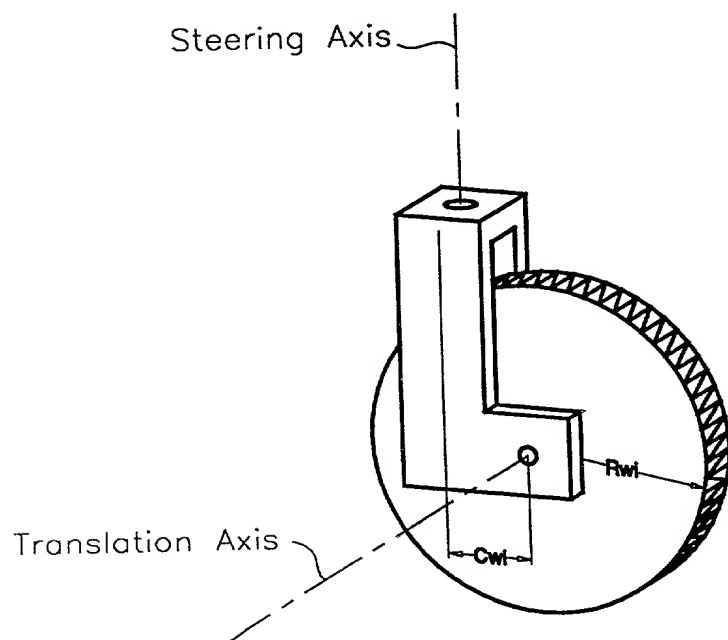


Fig. 1B

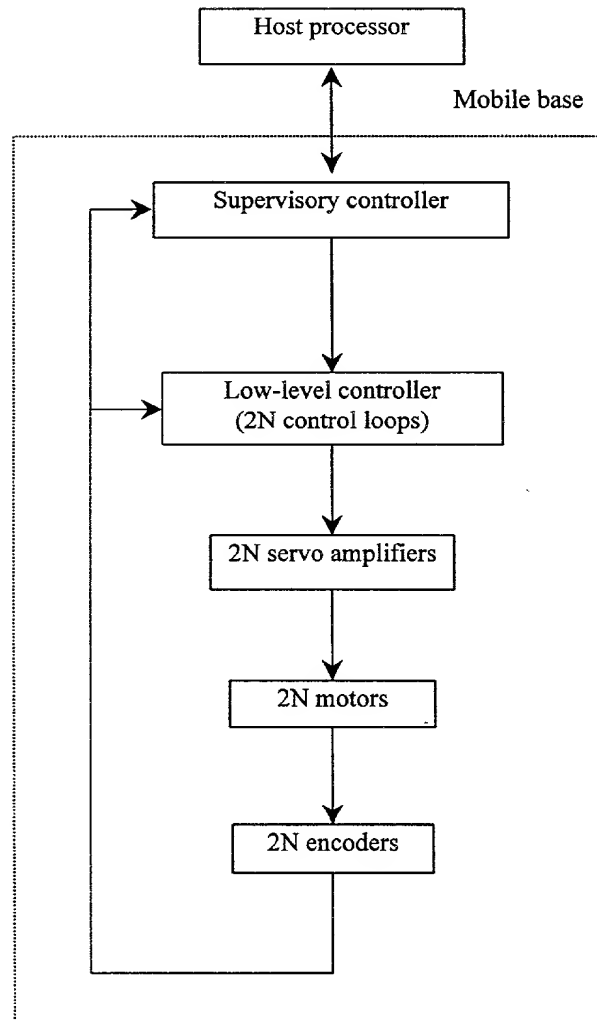


FIG. 2

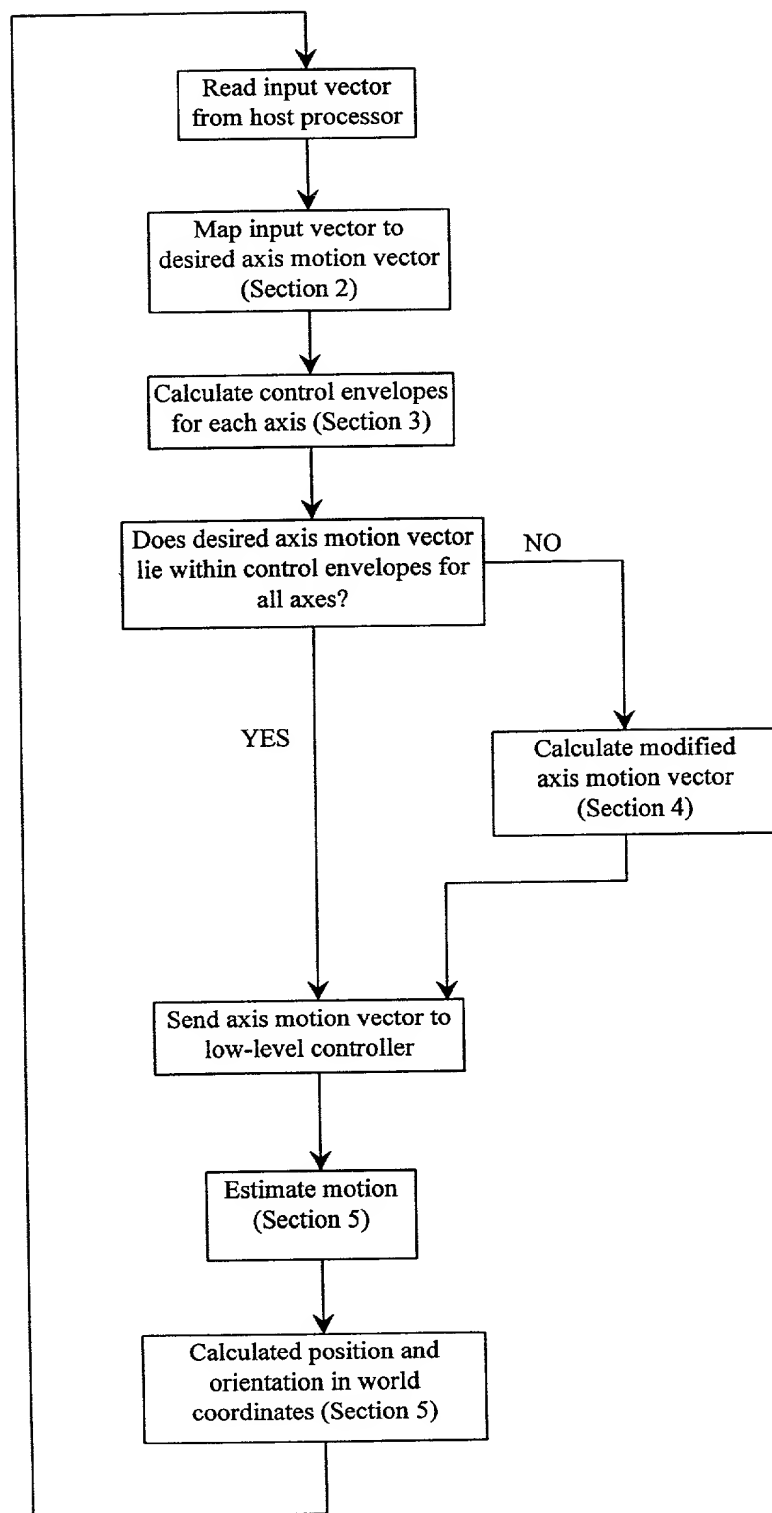


FIG. 3

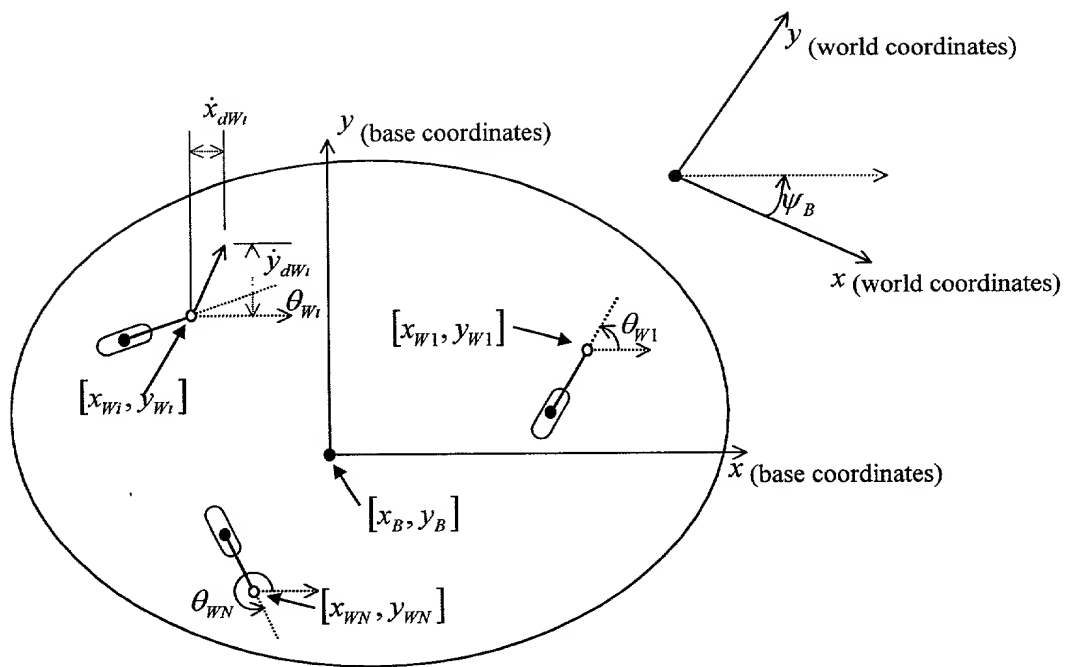


FIG. 4

**COMBINED DECLARATION FOR PATENT APPLICATION AND POWER OF ATTORNEY**

As a below named inventor, I hereby declare that:

My residence, post office address and citizenship are as stated below next to my name,

I believe I am the original, first and sole inventor (if only one name is listed below) or an original, first and joint inventor (if plural names are listed below) of the subject matter which is claimed and for which a patent is sought on the invention entitled

**METHOD AND APPARATUS FOR MOBILE ROBOT MOTION CONTROL**

the specification of which (check one) X is attached hereto or \_\_\_ was filed on \_\_\_ as Application No. \_\_\_ and was amended on \_\_\_ (if applicable).

I hereby state that I have reviewed and understand the contents of the above-identified specification, including the claims, as amended by any amendment referred to above.

I acknowledge the duty to disclose all information which is material to patentability as defined in 37 CFR § 1.56.

I hereby claim foreign priority benefits under 35 U.S.C. § 119(a)-(d) or § 365(b) of any foreign application(s) for patent or inventor's certificate, or § 365(a) of any PCT International application which designated at least one country other than the United States, listed below and have also identified below any foreign application for patent or inventor's certificate having a filing date before that of the application on which priority is claimed:

Prior Foreign Application(s)			Priority Claimed	
			Yes	No
PCT/US97/15605	PCT	05 September 1997	X	
Number	Country	Day/Month/Year Filed		
Number	Country	Day/Month/Year Filed		

I hereby claim the benefit under 35 U.S.C. § 119(e) of any United States provisional application(s) below.

60/025,406	September 6, 1996
Application Number	Filing Date

Application Number	Filing Date
--------------------	-------------

I hereby claim the benefit under 35 U.S.C. § 120 of any United States application(s), or § 365(c) of any PCT International application designating the United States, listed below and, insofar as the subject matter of each of the claims of this application is not disclosed in the prior United States application in the manner provided by the first paragraph of 35 U.S.C. § 112, I acknowledge the duty to disclose all information which is material to patentability as defined in 37 CFR § 1.56 which became available between the filing date of the prior application and the national or PCT international filing date of this application:

Application Number	Filing Date	Status: Patented, Pending, Abandoned
--------------------	-------------	--------------------------------------

Application Number	Filing Date	Status: Patented, Pending, Abandoned
--------------------	-------------	--------------------------------------

I HEREBY APPOINT THE FOLLOWING AS MY ATTORNEYS WITH FULL POWER OF SUBSTITUTION TO PROSECUTE THIS APPLICATION AND TRANSACT ALL BUSINESS IN THE PATENT OFFICE CONNECTED THEREWITH:

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Michael J. Pollock	29,098	Seong-Kun Oh*		William G. Goldman	42,590
Stephen M. Everett	30,050	Cameron A. King	41,897	J. Thomas McCarthy	22,420
Alfred A. Equitz	30,922	* Recognition under 37 CFR 10.9(b)		Joel G. Ackerman	24,307

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Telephone: 415/433-4150

I hereby declare that all statements made herein of my own knowledge are true and that all statements made on information and belief are believed to be true; and further that these statements were made with the knowledge that willful false statements and the like so made are punishable by fine or imprisonment or both, under 18 U.S.C. § 1001 and that such willful false statements may jeopardize the validity of the application or any patent issued thereon.

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